1) $f(x) = x + \frac{1}{x} = \frac{x^2+1}{x}$

1. $D(f) = \mathbb{R} - \{0\}$, $\quad \underset{0}{- \;\; \oplus \;\; +}$, $\quad f \cap x: \; y = 0 \Rightarrow 0 = \frac{x^2+1}{x} \Rightarrow \nexists$

$f \cap y: \; x = 0 \Rightarrow \nexists$

$f(-x) = -x - \frac{1}{x} = -(x + \frac{1}{x}) = -f(x) \Rightarrow$ f je lichá

2. $f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$, $\quad D(f') = \mathbb{R} - \{0\}$

$f'(x) = 0 \Rightarrow x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x_1 = 1, \; x_2 = -1$

$\nearrow \; \bullet \; \searrow \; \oplus \; \searrow \; \bullet \; \nearrow$

$+ \; -1 \; - \; 0 \; - \; 1 \; +$ $\qquad f(2) = 1 - \frac{1}{4} > 0$

$f(1) = 2$ ... $x = 1$ lokální minimum

$f(-1) = -2$ $\quad x = -1$ lokální maximum

rostoucí: $(-\infty; -1)$, $(1; \infty)$

klesající: $(-1; 0)$, $(0; 1)$

3. $f''(x) = \frac{2}{x^3}$, $\quad D(f'') = \mathbb{R} - \{0\}$; $\quad f''(x) \neq 0 \;\; \forall x \in D(f'') \Rightarrow$

$\Rightarrow$ inflexní body $\nexists$

$\cap \;\; \oplus \;\; \cup$

$- \;\; 0 \;\; +$

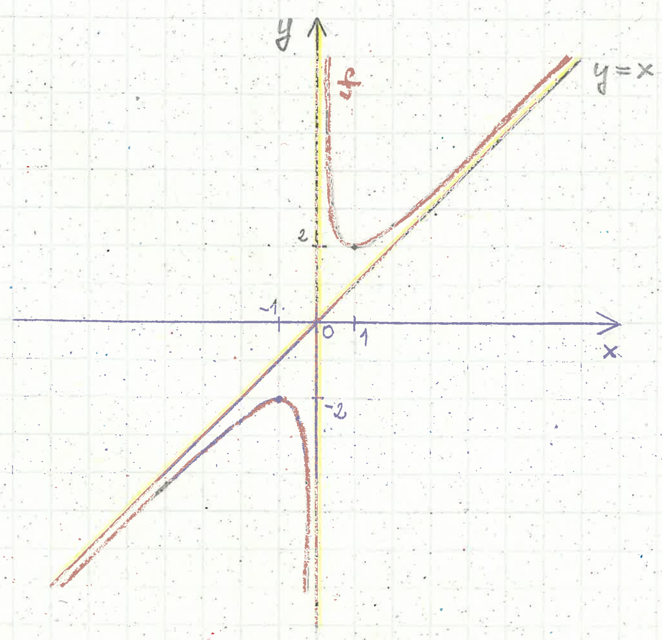
konvexní: $(0; \infty)$

konkávní: $(-\infty; 0)$

4. svislé: $\left.\begin{aligned} &\lim_{x \to 0^+} \frac{x^2+1}{x} = \frac{1}{0^+} = \infty \\ &\lim_{x \to 0^-} \frac{x^2+1}{x} = \frac{1}{0^-} = -\infty \end{aligned}\right\}$ $\quad x = 0$

šikmé: $\left.\begin{aligned} &a = \lim_{x \to \pm\infty} \frac{\frac{x^2+1}{x}}{x} = \lim_{x \to \pm\infty} \frac{x^2+1}{x^2} = \lim_{x \to \pm\infty} \frac{1 + \frac{1}{x^2}}{1} = 1 \\ &b = \lim_{x \to \pm\infty} (x + \frac{1}{x} - x) = \lim_{x \to \pm\infty} \frac{1}{x} = 0 \end{aligned}\right\}$ $\quad y = x$

5.



2) $f(x) = \dfrac{x^3}{3-x^2} = \dfrac{x^3}{(\sqrt{3}+x)\cdot(\sqrt{3}-x)}$

1. $D(f) = \mathbb{R} - \{-\sqrt{3}; \sqrt{3}\}$, $\quad \underset{}{+}\ \ \underset{-\sqrt{3}}{\oplus}\ \ -\ \ \underset{0}{\bullet}\ \ +\ \ \underset{\sqrt{3}}{\oplus}\ \ -$, $\quad f \cap x = f \cap y = \{[0;0]\}$

$f(-x) = \dfrac{(-x)^3}{3-(-x)^2} = -\dfrac{x^3}{3-x^2} = -f(x) \Rightarrow f$ je lichá

2. $f'(x) = \dfrac{3x^2(3-x^2) - x^3(-2x)}{(3-x^2)^2} = \dfrac{9x^2 - x^4}{(3-x^2)^2} = \dfrac{x^2(3-x)(3+x)}{(\sqrt{3}-x)^2(\sqrt{3}+x)^2}$ $\qquad D(f') = D(f)$

$\searrow\ -\ \underset{-3}{\bullet}\ \nearrow +\ \underset{-\sqrt{3}}{\circ}\ \nearrow +\ \underset{0}{\bullet}\ \nearrow +\ \underset{\sqrt{3}}{\circ}\ \nearrow +\ \underset{3}{\bullet}\ \searrow -$

LN: $-3; 3$
SN: $0; -\sqrt{3}; \sqrt{3}$

rostoucí: $(-3;-\sqrt{3}), (-\sqrt{3};\sqrt{3}), (\sqrt{3}, 3)$ $\qquad$ lokální maximum: $x = 3$, $f(3) = -\frac{9}{2}$
klesající: $(-\infty;-3), (3;\infty)$ $\qquad$ lokální minimum: $x = -3$, $f(-3) = \frac{9}{2}$

3. $f''(x) = \dfrac{(18x - 4x^3)\cdot(3-x^2)^2 - (9x^2-x^4)\cdot 2(3-x^2)(-2x)}{(3-x^2)^4} = \dfrac{6x^3+54x}{(3-x^2)^3} = \dfrac{6x(x^2+9)}{(\sqrt{3}-x)^3(\sqrt{3}+x)^3}$ $\qquad D(f'') = D(f)$

$\cup\ +\ \underset{-\sqrt{3}}{\circ}\ \cap\ -\ \underset{0}{\bullet}\ \cup\ +\ \underset{\sqrt{3}}{\circ}\ \cap\ -$ $\qquad$ LN: $0; -\sqrt{3}; \sqrt{3}$

konvexní: $(-\infty;-\sqrt{3}), (0;\sqrt{3})$ $\qquad$ inflexní bod: $x = 0$, $f(0) = 0$
konkávní: $(-\sqrt{3};0), (\sqrt{3};\infty)$

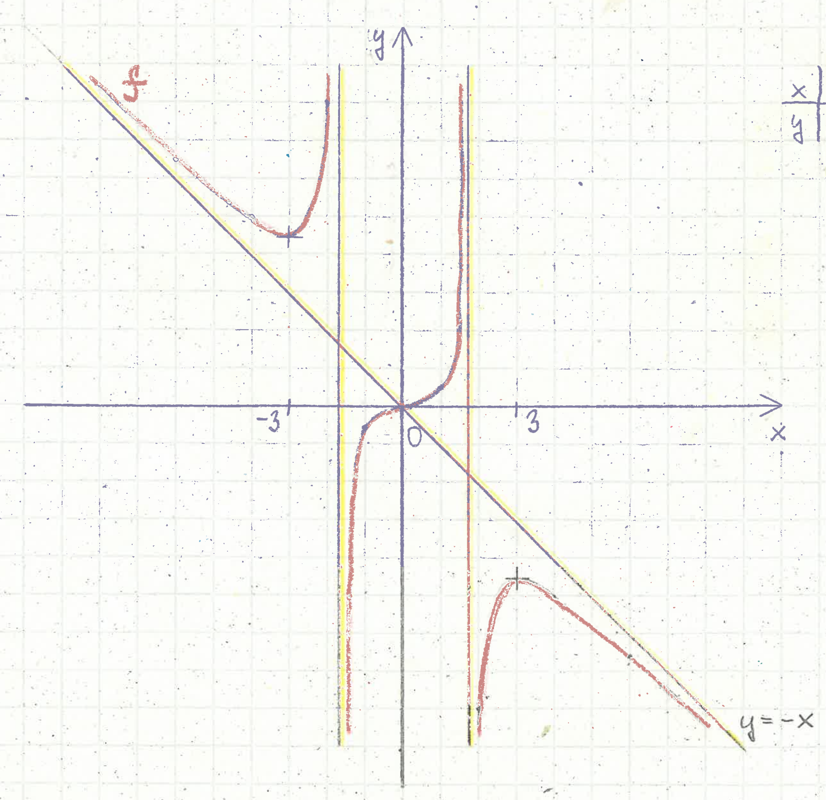
4. svislé: $\lim\limits_{x\to-\sqrt{3}^+} \dfrac{x^3}{3-x^2} = \dfrac{-3\sqrt{3}}{0^+} = -\infty$, $\lim\limits_{x\to-\sqrt{3}^-} \dfrac{x^3}{3-x^3} = \dfrac{-3\sqrt{3}}{0^-} = \infty \Rightarrow x = -\sqrt{3}$

$\lim\limits_{x\to\sqrt{3}^+} \dfrac{x^3}{3-x^2} = \dfrac{3\sqrt{3}}{0^-} = -\infty$, $\lim\limits_{x\to\sqrt{3}^-} \dfrac{x^3}{3-x^2} = \dfrac{3\sqrt{3}}{0^+} = \infty \Rightarrow x = \sqrt{3}$

šikmé: $a = \lim\limits_{x\to\pm\infty} \dfrac{\frac{x^3}{3-x^2}}{x} = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{3-x^2} = \lim\limits_{x\to\pm\infty} \dfrac{1}{\frac{3}{x^2}-1} = -1$

$b = \lim\limits_{x\to\pm\infty} \left(\dfrac{x^3}{3-x^2} + x\right) = \lim\limits_{x\to\pm\infty} \dfrac{3x}{3-x^2} = \lim\limits_{x\to\pm\infty} \dfrac{\frac{3}{x}}{\frac{3}{x^2}-1} = 0$ $\Rightarrow y = -x$

5.



x	1	-1	2	-2	4	-4
y	$\frac{1}{2}$	$-\frac{1}{2}$	-8	8	-4,9	4,9

3) $f(x) = \dfrac{e^x}{x+1}$

1. $D(f) = \mathbb{R} - \{-1\}$, $\quad \underset{-1}{-\ \circ\ +}$, $\quad f \cap x:\ y = 0 \Rightarrow \dfrac{e^x}{x+1} = 0 \Rightarrow \nexists$

$f \cap y:\ x = 0 \Rightarrow y = \dfrac{e^0}{0+1} = 1 \Rightarrow [0; 1]$

2. $f'(x) = \dfrac{e^x(x+1) - e^x}{(x+1)^2} = \dfrac{e^x(x+1-1)}{(x+1)^2} = \dfrac{x \cdot e^x}{(x+1)^2}$ $\quad D(f') = D(f)$

$\searrow\ -\ \circ_{-1}\ \searrow\ -\ \bullet_{0}\ \nearrow\ +$

LN: 0
SN: −1

rostoucí: $(0; \infty)$
klesající: $(-\infty; -1)$, $(-1; 0)$

lokální minimum: $x = 0$; $f(0) = 1$

3. $f''(x) = \dfrac{(e^x + xe^x)(x+1)^2 - xe^x \cdot 2(x+1)}{(x+1)^4} = \dfrac{xe^x + e^x + x^2e^x + xe^x - 2xe^x}{(x+1)^3} = \dfrac{e^x(1+x^2)}{(x+1)^3}$, $\quad D(f'') = D(f)$

$\cap\ -\ \circ_{-1}\ \cup\ +$

konvexní: $(-1; \infty)$
konkávní: $(-\infty; -1)$

$f''(x) \neq 0 \ \ \forall\ x \in D(f'') \Rightarrow$
$\Rightarrow$ inflexní body $\nexists$

4. svislé: $\lim\limits_{x \to -1^+} \dfrac{e^x}{x+1} = \dfrac{\frac{1}{e}}{0^+} = \infty$, $\quad \lim\limits_{x \to -1^-} \dfrac{e^x}{x+1} = \dfrac{\frac{1}{e}}{0^-} = -\infty \quad \Rightarrow \quad x = -1$

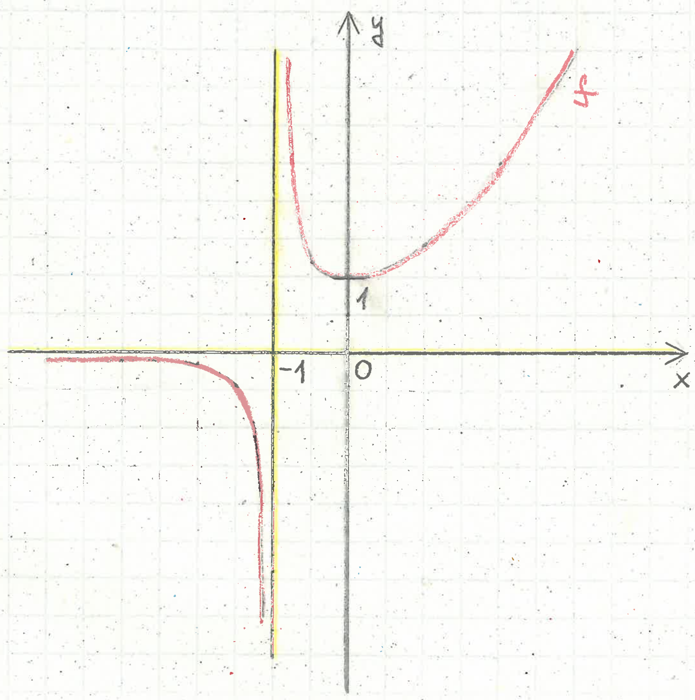
šikmé: $a_1 = \lim\limits_{x \to \infty} \dfrac{\frac{e^x}{x+1}}{x} = \lim\limits_{x \to \infty} \dfrac{e^x}{x^2+x} = \left[\dfrac{\infty}{\infty}\right] \overset{LP}{=} \lim\limits_{x \to \infty} \dfrac{e^x}{2x+1} \overset{LP}{=} \lim\limits_{x \to \infty} \dfrac{e^x}{2} = \infty$

$a_2 = \lim\limits_{x \to -\infty} \dfrac{\frac{e^x}{x+1}}{x} = \lim\limits_{x \to -\infty} \dfrac{e^x}{x^2+x} = \left[\dfrac{0}{\infty}\right] \overset{LP}{=} \lim\limits_{x \to -\infty} \dfrac{e^x}{2x+1} \overset{LP}{=} \lim\limits_{x \to -\infty} \dfrac{e^x}{2} = 0$

$b_2 = \lim\limits_{x \to -\infty} \dfrac{e^x}{x+1} = \left[\dfrac{0}{-\infty}\right] \overset{LP}{=} \lim\limits_{x \to -\infty} \dfrac{e^x}{1} = 0$

$\Rightarrow \quad y = 0$

5.



4) $f(x) = \frac{x^2}{e^x}$

1. $D(f) = \mathbb{R}$, $\quad + \; 0 \; +$ $\quad$ SN: 0 $\qquad f \cap x: \; y = 0 \Rightarrow x = 0 \qquad [0;0]$

$f \cap y: \; x = 0 \Rightarrow y = 0$

2. $f'(x) = \frac{2xe^x - x^2e^x}{e^{2x}} = \frac{2x - x^2}{e^x}$, $\quad D(f') = \mathbb{R}$

$f'(x) = 0 \Rightarrow 2x - x^2 = 0$

$x(2-x) = 0$

$x_1 = 0, \; x_2 = 2$

$\searrow \; \nearrow \; \searrow$

$- \; 0 \; + \; 2 \; -$ $\qquad$ LN: 0; 2

rostoucí: $(0;2)$ $\qquad$ lokální minimum: $x = 0$, $f(0) = 0$

klesající: $(-\infty;0), (2;\infty)$ $\qquad$ lokální maximum: $x = 2$, $f(2) = \frac{4}{e^2} \doteq 0{,}54$

3. $f''(x) = \frac{(2-2x)e^x - (2x-x^2)\cdot e^x}{e^{2x}} = \frac{2 - 2x - 2x + x^2}{e^x} = \frac{x^2 - 4x + 2}{e^x}$, $\quad D(f'') = \mathbb{R}$

$f''(x) = 0 \Rightarrow x^2 - 4x + 2 = 0$

$D = 16 - 8 = 8$

$x_{1,2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

$\cup \; \cap \; \cup$

$+ \; 2-\sqrt{2} \; - \; 2+\sqrt{2} \; +$

konvexní: $(-\infty; 2-\sqrt{2}), (2+\sqrt{2}; \infty)$ $\qquad$ inflexní body: $x = 2-\sqrt{2} \doteq 0{,}6 \quad f(2-\sqrt{2}) \doteq 0{,}2$

konkávní: $(2-\sqrt{2}; 2+\sqrt{2})$ $\qquad x = 2+\sqrt{2} \doteq 3{,}4 \quad f(2+\sqrt{2}) \doteq 0{,}38$

4. svislé ∄

šikmé:

$$a_1 = \lim_{x\to\infty} \frac{\frac{x^2}{e^x}}{x} = \lim_{x\to\infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty}\right] \overset{LP}{=} \lim_{x\to\infty} \frac{1}{\underbrace{e^x}_{\to\infty}} = 0$$

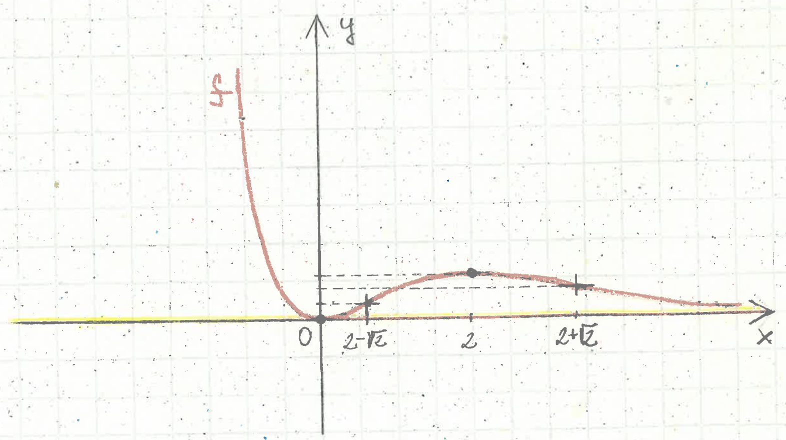
$$b_1 = \lim_{x\to\infty} \frac{x^2}{e^x} = \left[\frac{\infty}{\infty}\right] \overset{LP}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{LP}{=} \lim_{x\to\infty} \frac{2}{\underbrace{e^x}_{\to\infty}} = 0$$

$\Rightarrow y = 0$

$$a_2 = \lim_{x\to-\infty} \frac{\frac{x^2}{e^x}}{x} = \lim_{x\to-\infty} \frac{x}{e^x} = \left[\frac{-\infty}{0}\right] = \lim_{x\to-\infty} x \cdot \lim_{x\to-\infty} \frac{1}{\underbrace{e^x}_{\to 0^+}} = -\infty \cdot \infty = -\infty$$

$$b_2 = \lim_{x\to-\infty} \frac{x^2}{e^x} = \left[\frac{\infty}{0}\right] = \lim_{x\to-\infty} x^2 \cdot \lim_{x\to-\infty} \frac{1}{e^x} = (-\infty)^2 \cdot \infty = \infty$$

5.



5) $f(x) = \frac{\ln x}{x}$

1. $D(f) = (0; \infty)$, $\ln x = 0 \Rightarrow x = 1$, $f \cap x$: $y = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$ $[1; 0]$

$f \cap y$: $x = 0 \Rightarrow \nexists$

$0 \quad - \quad 1 \quad +$

$f(2) = \frac{\ln 2}{2} > 0$

2. $f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$, $D(f') = D(f)$

$f'(x) = 0 \Rightarrow 1 - \ln x = 0$

$\ln x = 1$

$x = e \doteq 2{,}7$

$0 \quad + \nearrow \quad e \quad - \searrow$

LN: e

$f'(1) = \frac{1 - \ln 1}{1} = 1 > 0$

rostoucí: $(0; e)$

klesající: $(e; \infty)$

lokální maximum: $x = e$, $f(e) = \frac{1}{e} \doteq 0{,}37$

3. $f''(x) = \frac{-\frac{1}{x} \cdot x^2 - (1 - \ln x) \cdot 2x}{x^4} = \frac{-1 - 2 + 2\ln x}{x^3} = \frac{2\ln x - 3}{x^3}$, $D(f'') = D(f)$

$f''(x) = 0 \Rightarrow 2\ln x - 3 = 0$

$\ln x = \frac{3}{2}$

$x = \sqrt{e^3} \doteq 4{,}5$

$0 \quad - \cap \quad \sqrt{e^3} \quad + \cup$

LN: $\sqrt{e^3}$

$f''(1) = \frac{2\ln 1 - 3}{1} = -3 < 0$

konvexní: $(\sqrt{e^3}; \infty)$

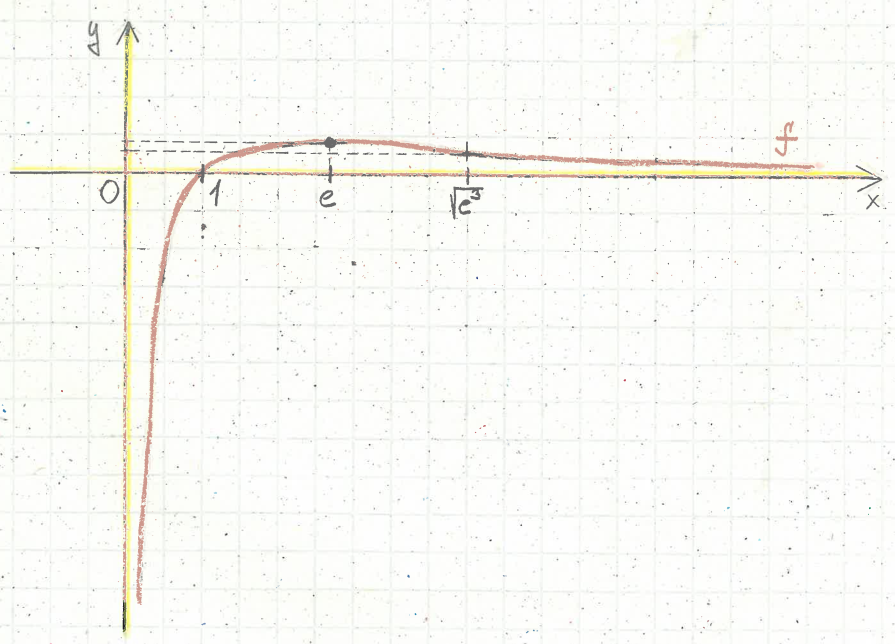
konkávní: $(0; \sqrt{e^3})$

inflexní bod: $x = \sqrt{e^3}$; $f(\sqrt{e^3}) = \frac{3}{2\sqrt{e^3}} \doteq 0{,}3$

4. svislé: $\lim_{x \to 0^+} \frac{\ln x}{x} = \left[\frac{-\infty}{0}\right] = \lim_{x \to 0^+} \ln x \cdot \lim_{x \to 0^+} \frac{1}{x} = -\infty \cdot \infty = -\infty \Rightarrow x = 0$

šikmé:
$$\left.\begin{aligned} a &= \lim_{x \to \infty} \frac{\frac{\ln x}{x}}{x} = \lim_{x \to \infty} \frac{\ln x}{x^2} = \left[\frac{\infty}{\infty}\right] \overset{LP}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{2x} = \lim_{x \to \infty} \frac{1}{2x^2} = 0 \\ b &= \lim_{x \to \infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right] \overset{LP}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x} = 0 \end{aligned}\right\} \Rightarrow y = 0$$

5.



6) $f(x) = \frac{1}{\ln x} = \ln^{-1} x$

1. $D(f) = (0;\infty) - \{1\}$, $\ln x \neq 0$, $x \neq 1$, $\underset{0}{\circ} \; - \; \underset{1}{\circ} \; +$, $f \cap x$, $f \cap y$ $\nexists$

2. $f'(x) = -\ln^{-2} x \cdot \frac{1}{x} = -\frac{1}{x \cdot \ln^2 x}$, $D(f') = D(f)$ $\underset{0}{\circ} \; \searrow - \; \underset{1}{\circ} \; \searrow -$ SN: 1, $f'(2) = -\frac{1}{2\ln^2 2} < 0$

klesající: $(0;1)$, $(1;\infty)$, rostoucí: $\emptyset$, extrémy $\nexists$

3. $f''(x) = -\frac{0 - (\ln^2 x + x \cdot 2\ln x \cdot \frac{1}{x})}{x^2 \cdot \ln^4 x} = \frac{2 + \ln x}{x^2 \cdot \ln^3 x}$ $D(f'') = D(f)$

$f''(0) = 0 \Rightarrow 2 + \ln x = 0$

$\ln x = -2$

$x = e^{-2} \doteq 0{,}14$

$\underset{0}{\circ} \; \overset{\cup}{+} \; \underset{e^{-2}}{\bullet} \; \overset{\cap}{-} \; \underset{1}{\circ} \; \overset{\cup}{+}$ LN: e^{-2}; 1

konvexní: $(0; e^{-2})$, $(1;\infty)$ inflexní bod: $x = e^{-2}$, $f(e^{-2}) = \frac{1}{\ln e^{-2}} = -\frac{1}{2}$

konkávní: $(e^{-2}; 1)$

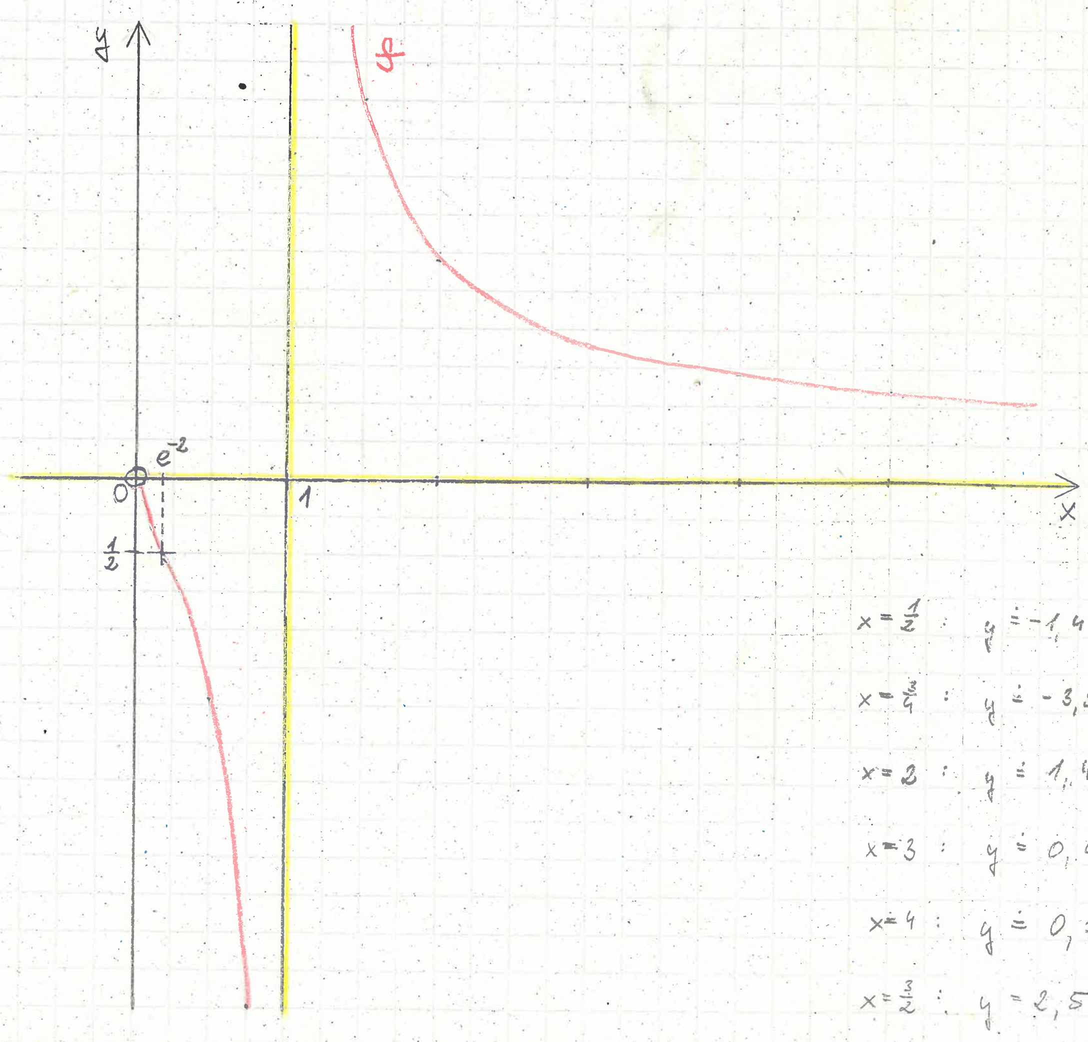
4. svislé: $\lim_{x\to 0^+} \frac{1}{\ln x} = \frac{1}{-\infty} = 0$

$\lim_{x\to 1^+} \frac{1}{\ln x} = \frac{1}{0^+} = \infty$, $\lim_{x\to 1^-} \frac{1}{\ln x} = \frac{1}{0^-} = -\infty \Rightarrow x = 1$

šikmé: $a = \lim_{x\to\infty} \frac{\frac{1}{\ln x}}{x} = \lim_{x\to\infty} \frac{1}{x \cdot \ln x} = \frac{1}{\infty \cdot \infty} = 0$

$b = \lim_{x\to\infty} \frac{1}{\ln x} = \frac{1}{\infty} = 0$ $\Rightarrow$ $y = 0$

5.



$x = \frac{1}{2}$: $y \doteq -1{,}44$

$x = \frac{3}{4}$: $y \doteq -3{,}5$

$x = 2$: $y \doteq 1{,}44$

$x = 3$: $y \doteq 0{,}9$

$x = 4$: $y \doteq 0{,}7$

$x = \frac{3}{2}$: $y \doteq 2{,}5$

7) $f(x) = \arcsin \dfrac{1-x^2}{1+x^2}$

1. $1-x^2 \leq 1+x^2 \quad \forall\, x \in \mathbb{R} \;\Rightarrow\; \dfrac{1-x^2}{1+x^2} \in \langle -1; 1 \rangle \quad \forall\, x \in \mathbb{R} \;\Rightarrow\; D(f) = \mathbb{R}$

$\arcsin \dfrac{1-x^2}{1+x^2} = 0 \;\Rightarrow\; \dfrac{1-x^2}{1+x^2} = 0 \;\Rightarrow\; x_1 = -1, \; x_2 = 1$

$$\begin{array}{ccccc} - & & + & & - \\ \hline & -1 & & 1 & \end{array}$$

$f(0) = \arcsin 1 = \frac{\pi}{2} > 0$

$f \cap x: \; x_1 = -1, \; f(-1) = \arcsin 0 = 0 \qquad [-1; 0]$

$ x_2 = 1, \; f(1) = \arcsin 0 = 0 \qquad [1; 0]$

$f \cap y: \; x = 0 \Rightarrow y = \arcsin 1 = \frac{\pi}{2} \qquad [0; \frac{\pi}{2}]$

$f(-x) = \arcsin \dfrac{1-(-x)^2}{1+(-x)^2} = \arcsin \dfrac{1-x^2}{1+x^2} = f(x) \;\Rightarrow\;$ f je sudá

2. $f'(x) = \dfrac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \dfrac{-2x(1+x^2)-(1-x^2)\cdot 2x}{(1+x^2)^2} = \sqrt{\dfrac{(1+x^2)^2}{1+2x^2+x^4-1+2x^2-x^4}} \cdot \dfrac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} =$

$= \sqrt{\dfrac{(1+x^2)^2}{4x^2}} \cdot \dfrac{-4x}{(1+x^2)^2} = \dfrac{1+x^2}{2|x|} \cdot \dfrac{-4x}{(1+x^2)^2} = -\dfrac{2x}{|x|(1+x^2)} \qquad D(f') = \mathbb{R} - \{0\}$

$$\begin{array}{ccc} \nearrow & & \searrow \\ \hline + & 0 & - \end{array}$$

$f'(1) = -\dfrac{2}{1 \cdot 2} < 0 \;, \quad f'(-1) = -\dfrac{-2}{2 \cdot 2} > 0$

rostoucí: $(-\infty; 0)$ lokální maximum může být v $x = 0$, $f(0) = \frac{\pi}{2} \doteq 1{,}6$

klesající: $(0; \infty)$

3. $f''(x) = \left[-\dfrac{2}{(1+x^2)}\right]' = \dfrac{4x}{(1+x^2)^2}, \quad x > 0 \qquad D(f'') = D(f') = \mathbb{R} - \{0\}$

$= \left[\dfrac{2}{(1+x^2)}\right]' = -\dfrac{4x}{(1+x^2)^2}, \quad x < 0$

$$\begin{array}{ccc} \cup & & \cup \\ \hline + & 0 & + \end{array}$$

$f''(1) = \frac{+}{+} > 0$

$f''(-1) = -\frac{-}{+} > 0$

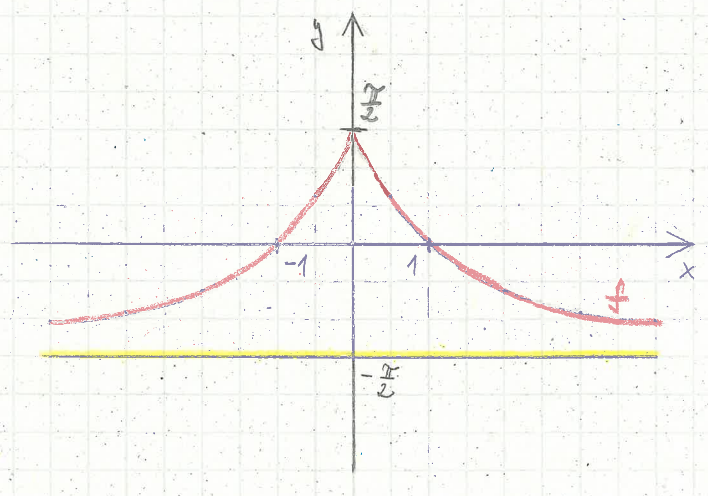
konvexní: $(-\infty; 0), (0; \infty)$, konkávní: $\emptyset$, inflexní body $\nexists$

4. svislé $\nexists$

šikmé: $a = \lim\limits_{x \to \pm\infty} \dfrac{\arcsin \frac{1-x^2}{1+x^2}}{x} = \dfrac{\arcsin\left(\lim\limits_{x\to\pm\infty} \frac{\frac{1}{x^2}-1}{\frac{1}{x^2}+1}\right)}{\lim\limits_{x\to\pm\infty} x} = \dfrac{\arcsin(-1)}{\pm\infty} = \dfrac{-\frac{\pi}{2}}{\pm\infty} = 0$

$b = \lim\limits_{x \to \pm\infty} \arcsin \dfrac{1-x^2}{1+x^2} = \arcsin(-1) = -\dfrac{\pi}{2} \;\Rightarrow\; y = -\dfrac{\pi}{2}$

5.



8) $f(x) = \frac{(x-1)^2}{x+2}$

1. $D(f) = \mathbb{R} - \{-2\}$

$- \underset{-2}{\circ} + \underset{1}{\bullet} +$

$f \cap x: \ (x-1)^2 = 0 \Rightarrow x = 1 \Rightarrow [1;0]$

$f \cap y: \ x = 0 \Rightarrow y = \frac{1}{2} \Rightarrow [0;\frac{1}{2}]$

2. $f'(x) = \frac{2(x-1)(x+2) - (x-1)^2}{(x+2)^2} = \frac{(x-1)(2x+4-x+1)}{(x+2)^2} = \frac{(x-1)(x+5)}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2}$ $\quad D(f') = D(f)$

$\nearrow \ + \ \underset{-5}{\bullet} \ - \ \searrow \ \underset{-2}{\circ} \ - \ \searrow \ \underset{1}{\bullet} \ + \ \nearrow$

LN: 1; −5

SN: −2

$f'(0) = \frac{-5}{4} < 0$

rostoucí: $(-\infty; -5)$; $(1; \infty)$

klesající: $(-5; -2)$; $(-2; 1)$

lokální maximum: $x = -5$, $f(-5) = -12$

lokální minimum: $x = 1$, $f(1) = 0$

3. $f''(x) = \frac{(2x+4)(x+2)^2 - (x^2+4x-5)\cdot 2(x+2)}{(x+2)^4} = \frac{2x^2+4x+4x+8-2x^2-8x+10}{(x+2)^3} = \frac{18}{(x+2)^3}$ $\quad D(f'') = D(f)$

$\cap \ - \ \underset{-2}{\circ} \ + \ \cup$

konvexní: $(-2; \infty)$

konkávní: $(-\infty; -2)$

$f''(x) \neq 0 \ \forall \ x \in D(f'') \Rightarrow$

$\Rightarrow$ inflexní body $\nexists$

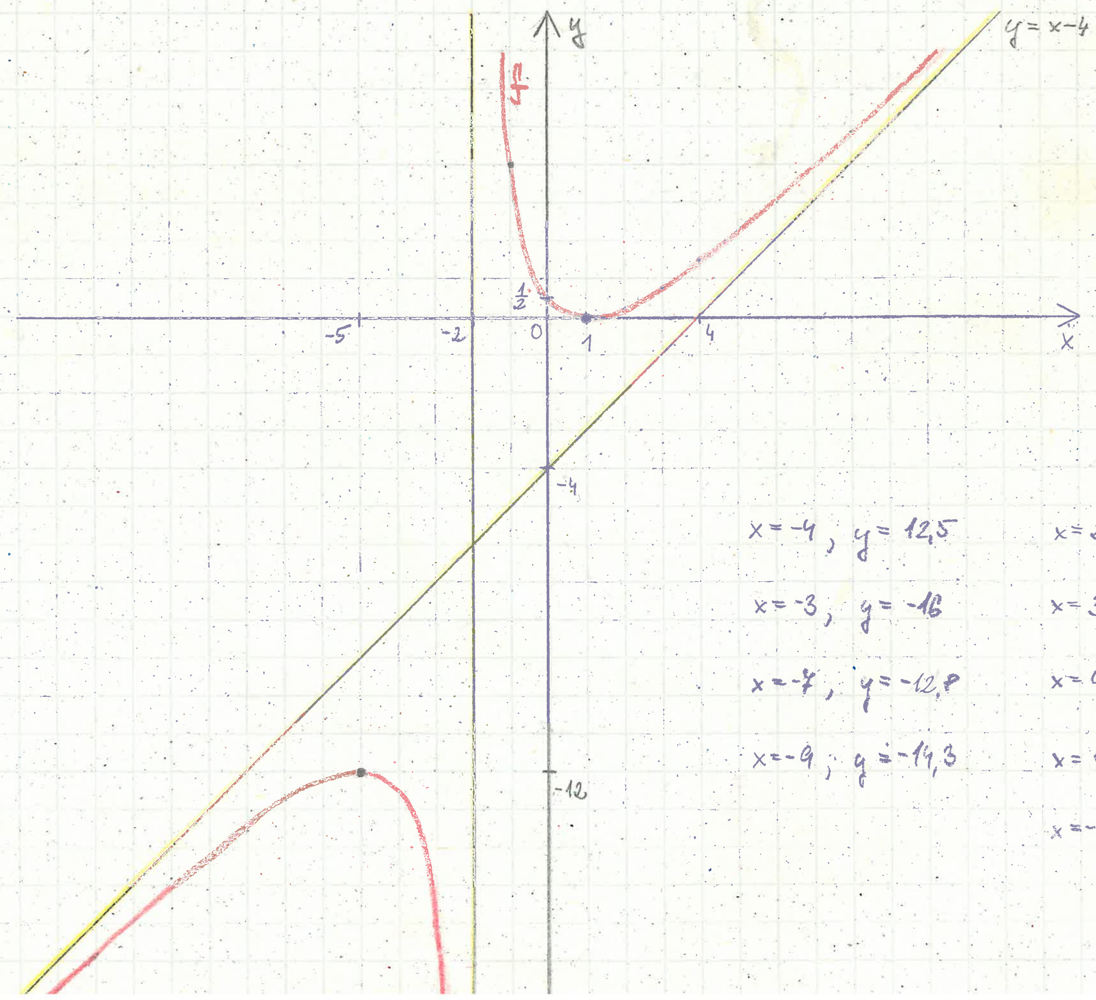
4. svislé: $\lim\limits_{x \to -2^+} \frac{(x-1)^2}{x+2} = \frac{9}{0^+} = \infty$, $\lim\limits_{x \to -2^-} \frac{(x-1)^2}{x+2} = \frac{9}{0^-} = -\infty \Rightarrow$ $x = -2$

šikmé:

$$\left.\begin{aligned} a &= \lim_{x \to \pm\infty} \frac{\frac{(x-1)^2}{x+2}}{x} = \lim_{x \to \pm\infty} \frac{x^2-2x+1}{x^2+2x} = \lim_{x \to \pm\infty} \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{1 + \frac{2}{x}} = 1 \\ b &= \lim_{x \to \pm\infty} \left(\frac{(x-1)^2}{x+2} - x \right) = \lim_{x \to \pm\infty} \frac{x^2-2x+1-x^2-2x}{x+2} = \lim_{x \to \pm\infty} \frac{-4x+1}{x+2} = \lim_{x \to \pm\infty} \frac{-4+\frac{1}{x}}{1+\frac{2}{x}} = -4 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow$ $y = x - 4$

5.



$x = -4$, $y = 12{,}5$

$x = -3$, $y = -16$

$x = -7$, $y = -12{,}8$

$x = -9$; $y = -14{,}3$

$x = 2$, $y = \frac{1}{4}$

$x = 3$, $y = 0{,}8$

$x = 4$, $y = \frac{3}{2}$

$x = 8$, $y = 4{,}9$

$x = -1$, $y = 4$

9) $f(x) = \dfrac{x}{1+x^2}$

1. $D(f) = \mathbb{R}$

$$\begin{array}{ccc} - & & + \\ \hline & 0 & \end{array}$$

$f \cap x: \; y = 0 \Rightarrow x = 0 \Rightarrow [0;0]$
$f \cap y: \; x = 0 \Rightarrow y = 0 \Rightarrow$ 〃

$$f(-x) = \frac{-x}{1+(-x)^2} = -\frac{x}{1+x^2} = -f(x) \Rightarrow f \text{ je lichá}$$

2. $$f'(x) = \frac{1+x^2 - x\cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = \frac{(1+x)(1-x)}{(1+x^2)^2}$$ $D(f') = D(f) = \mathbb{R}$

$$\begin{array}{ccccc} \searrow & & \nearrow & & \searrow \\ \hline - & -1 & + & 1 & - \end{array}$$

rostoucí: $(-1;1)$
klesající: $(-\infty;-1), (1;\infty)$

lokální maximum: $x = 1$, $f(1) = \frac{1}{2}$
lokální minimum: $x = -1$, $f(-1) = -\frac{1}{2}$

3. $$f''(x) = \frac{-2x(1+x^2)^2 - (1-x^2)\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4} = \frac{-2x-2x^3-4x+4x^3}{(1+x^2)^3} = \frac{2x^3-6x}{(1+x^2)^3} =$$
$$= \frac{2x(x^2-3)}{(1+x^2)^3} = \frac{2x(x+\sqrt{3})(x-\sqrt{3})}{(1+x^2)^3}$$

$$\begin{array}{ccccccc} \cap & & \cup & & \cap & & \cup \\ \hline - & -\sqrt{3} & + & 0 & - & \sqrt{3} & + \end{array}$$

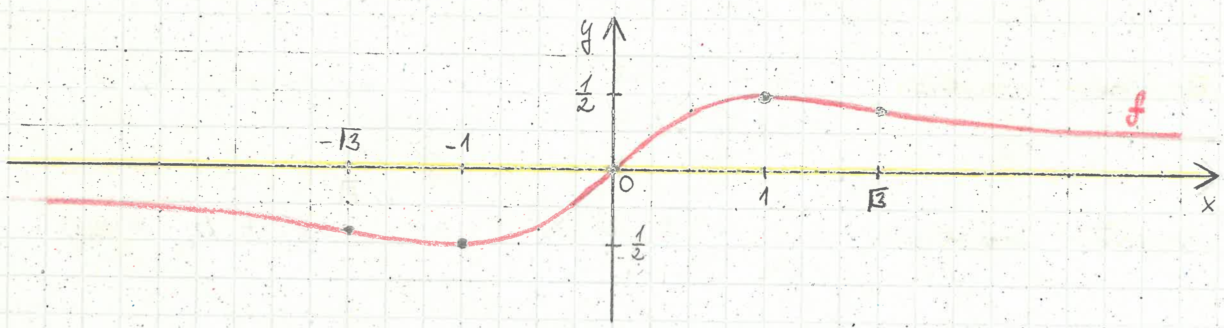
$f(1) = \frac{2-6}{(1+1)^3} < 0$

konvexní: $(-\sqrt{3};0), (\sqrt{3};\infty)$
konkávní: $(-\infty;-\sqrt{3}), (0;\sqrt{3})$

inflexní body: $x = 0$, $f(0) = 0$
$x = -\sqrt{3}$, $f(-\sqrt{3}) = -\frac{\sqrt{3}}{4}$
$x = \sqrt{3} \doteq 1{,}73$, $f(\sqrt{3}) = \frac{\sqrt{3}}{4} \doteq 0{,}43$

4. svislé ∄
šikmé: $$\left.\begin{aligned} a &= \lim_{x\to\pm\infty} \frac{\frac{x}{1+x^2}}{x} = \lim_{x\to\pm\infty} \frac{1}{1+x^2} = \lim_{x\to\pm\infty} \frac{\frac{1}{x^2}}{\frac{1}{x^2}+1} = 0 \\ b &= \lim_{x\to\pm\infty} \frac{x}{1+x^2} = \lim_{x\to\pm\infty} \frac{\frac{1}{x}}{\frac{1}{x^2}+1} = 0 \end{aligned}\right\} \Rightarrow y = 0$$

5.



$x = \frac{1}{2}$, $y = \frac{2}{5} = 0{,}4$

$x = 3$, $y = \frac{3}{10} = 0{,}3$